Rossmoyne Senior High School

WA Exams Practice Paper D, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 1
Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

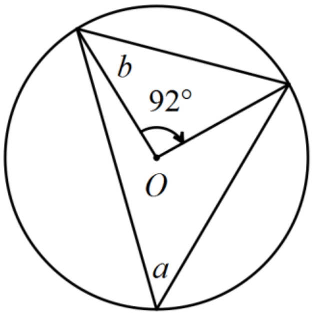
This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

In the following diagrams, O is the centre of the circle shown.

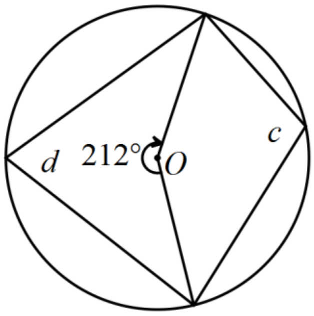
(a) Determine the values of a and b. (2 marks)



$a = 92 \div 2$
$= 46°$

$b = \dfrac{180 - 92}{2}$
$= 44°$

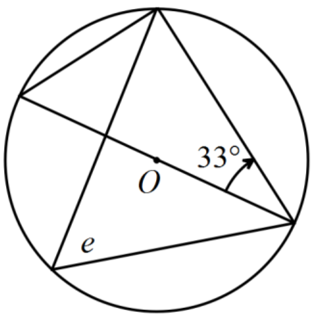
(b) Determine the values of c and d. (2 marks)



$c = 212 \div 2$
$= 106°$

$d = 180 - 106$
$= 74°$

(c) Determine the value of e. (1 mark)



$e = 90 - 33$
$= 57°$

Question 2 **(7 marks)**

Three points are given by $A(1,\ 2)$, $B(4,-2)$ and $C(p,\ 4)$.

(a) Determine a unit vector parallel to the line through AB. (2 marks)

$$\overrightarrow{AB}=\begin{bmatrix}4\\-2\end{bmatrix}-\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}3\\-4\end{bmatrix}$$

$$\left|\overrightarrow{AB}\right|=\sqrt{3^2+(-4)^2}=5$$

Unit vector is $\frac{1}{5}\begin{bmatrix}3\\-4\end{bmatrix}$

(b) Show that A, B and $D(19,-22)$ are collinear. (2 marks)

$$\overrightarrow{AD}=\begin{bmatrix}19-1\\-22-2\end{bmatrix}=\begin{bmatrix}18\\-24\end{bmatrix}=6\times\overrightarrow{AB}$$

Hence collinear as parallel vectors with A in common.

(c) The lines through AB and BC are perpendicular.

(i) Write down the vector $\overrightarrow{BC}$. (1 mark)

$$\overrightarrow{BC}=\begin{bmatrix}p\\4\end{bmatrix}-\begin{bmatrix}4\\-2\end{bmatrix}=\begin{bmatrix}p-4\\6\end{bmatrix}$$

(ii) Evaluate the dot product of $\overrightarrow{AB}$ and $\overrightarrow{BC}$. (1 mark)

$$\begin{bmatrix}3\\-4\end{bmatrix}\bullet\begin{bmatrix}p-4\\6\end{bmatrix}=3p-12-24$$

(iii) Show that $p=12$. (1 mark)

$$3p-12-24=0$$
$$p=12$$

Question 3 **(6 marks)**

A true statement is 'if a quadrilateral is a square then the lengths of both diagonals are equal in length'.

(a) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

> If the lengths of both diagonals of a quadrilateral are equal then it is a square.
>
> False - it could be a rectangle.

(b) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

> If the lengths of both diagonals of a quadrilateral are not equal then it is not a square.
>
> True – contrapositive statements are always true.

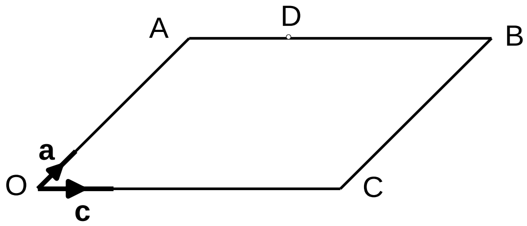
(c) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

> If a quadrilateral is not a square then the lengths of both diagonals are not equal in length.
>
> False - it could be a rectangle.

Question 4 **(6 marks)**

In the parallelogram OABC below, $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OC} = 6\mathbf{c}$.

D is a point on AB such that AD:DB $= 1:2$.



(a) Express the following in terms of $\mathbf{a}$ and/or $\mathbf{c}$.

(i) $\overrightarrow{AC}$ (1 mark)

$6\mathbf{c} - 4\mathbf{a}$

(ii) $\overrightarrow{AD}$ (1 mark)

$2\mathbf{c}$

(iii) $\overrightarrow{DC}$ (1 mark)

$4\mathbf{c} - 4\mathbf{a}$

(b) M is the midpoint of AC. Express $\overrightarrow{MD}$ in terms of $\mathbf{a}$ and/or $\mathbf{c}$. (3 marks)

$$\begin{aligned}
\overrightarrow{OM} &= \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AC} \\
&= 2\mathbf{a} + 3\mathbf{c} \\
\overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \\
&= 4\mathbf{a} + 2\mathbf{c} \\
\overrightarrow{MD} &= \overrightarrow{OD} - \overrightarrow{OM} \\
&= 4\mathbf{a} + 2\mathbf{c} - 2\mathbf{a} - 3\mathbf{c} \\
&= 2\mathbf{a} - \mathbf{c}
\end{aligned}$$

Question 5 **(7 marks)**

(a) Prove that ${}^{n}C_{r} = \frac{n}{r} \times {}^{n-1}C_{r-1}$. (3 marks)

$$\begin{aligned}
{}^{n}C_{r} &= \frac{n!}{r! \times (n-r)!} \\
&= \frac{n \times (n-1)!}{r \times (r-1)! \times (n-1-r+1)!} \\
&= \frac{n}{r} \times \frac{(n-1)!}{(r-1)! \times \left((n-1)-(r-1)\right)!} \\
&= \frac{n}{r} \times {}^{n-1}C_{r-1}
\end{aligned}$$

(b) Given that ${}^{14}C_{5} = 2002$ and ${}^{15}C_{5} = 3003$, determine

(i) ${}^{15}C_{6}$. (2 marks)

$$\begin{aligned}
{}^{15}C_{6} &= \frac{15}{6} \times {}^{14}C_{5} \\
&= \frac{5}{2} \times 2002 \\
&= 5005
\end{aligned}$$

(ii) ${}^{14}C_{4}$. (2 marks)

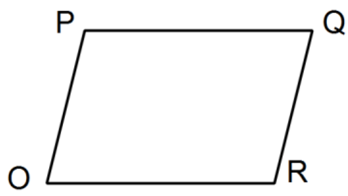
$$\begin{aligned}
{}^{15}C_{5} &= \frac{15}{5} \times {}^{14}C_{4} \\
3003 &= 3 \times {}^{14}C_{4} \\
{}^{14}C_{4} &= 1001
\end{aligned}$$

Question 6 **(8 marks)**

(a) If $\mathbf{a}$ and $\mathbf{b}$ are two non-zero vectors such that $|\mathbf{a}+\mathbf{b}|=3$, simplify $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}+\mathbf{b})$. (1 mark)

$$\begin{aligned}(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}+\mathbf{b}) &= |\mathbf{a}+\mathbf{b}|\bullet|\mathbf{a}+\mathbf{b}|\cos(0)\\ &= 3\times 3\times 1\\ &= 9\end{aligned}$$

(b) Let $OPQR$ be a parallelogram with sides $\overrightarrow{OP}=\mathbf{p}$ and $\overrightarrow{OR}=\mathbf{r}$.



(i) Determine expressions, in terms of $\mathbf{p}$ and $\mathbf{r}$, for $\overrightarrow{OQ}$ and $\overrightarrow{PR}$. (2 marks)

$$\overrightarrow{OQ}=\mathbf{p}+\mathbf{r}$$

$$\overrightarrow{PR}=\mathbf{r}-\mathbf{p}$$

(ii) Determine an expression, in terms of $\mathbf{p}$ and $\mathbf{r}$, for the sum of the squares of the lengths of the diagonals of $OPQR$. (1 mark)

$$sum=|\mathbf{p}+\mathbf{r}|^2+|\mathbf{r}-\mathbf{p}|^2$$

(iii) Determine an expression, in terms of $\mathbf{p}$ and $\mathbf{r}$, for the sum of the squares of the lengths of all four sides of $OPQR$. (1 mark)

$$sum=2|\mathbf{p}|^2+2|\mathbf{r}|^2$$

(iv) Prove that the two expressions in (ii) and (iii) are equal. (3 marks)

$$
\begin{aligned}
|\mathbf{p}+\mathbf{r}|^2 + |\mathbf{r}-\mathbf{p}|^2 &= (\mathbf{p}+\mathbf{r}) \bullet (\mathbf{p}+\mathbf{r}) + (\mathbf{r}-\mathbf{p}) \bullet (\mathbf{r}-\mathbf{p}) \\
&= \mathbf{p} \bullet \mathbf{p} + \mathbf{r} \bullet \mathbf{p} + \mathbf{r} \bullet \mathbf{p} + \mathbf{r} \bullet \mathbf{r} + \mathbf{p} \bullet \mathbf{p} - \mathbf{r} \bullet \mathbf{p} - \mathbf{r} \bullet \mathbf{p} + \mathbf{r} \bullet \mathbf{r} \\
&= |\mathbf{p}|^2 + |\mathbf{r}|^2 + |\mathbf{p}|^2 + |\mathbf{r}|^2 \\
&= 2|\mathbf{p}|^2 + 2|\mathbf{r}|^2
\end{aligned}
$$

Question 7 **(6 marks)**

(a) Relative to a lighthouse, a ship has position vector $(4\mathbf{i}+7\mathbf{j})$ km. Relative to an observer on an island, the ship has position vector $(13\mathbf{i}-5\mathbf{j})$ km. Determine the exact distance of the observer on the island from the lighthouse. (3 marks)

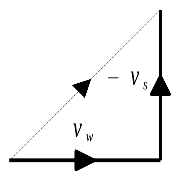
$$\mathbf{r}_s = \begin{bmatrix} 4 \\ 7 \end{bmatrix} \qquad {}_s\mathbf{r}_o = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

$$\mathbf{r}_o = \mathbf{r}_s - {}_s\mathbf{r}_o = \begin{bmatrix} 4 \\ 7 \end{bmatrix} - \begin{bmatrix} 13 \\ -5 \end{bmatrix} = \begin{bmatrix} -9 \\ 12 \end{bmatrix}$$

$|\mathbf{r}_o| = 15$ km

(b) The wind is blowing at 9 km/h from the west and the ship is moving due south at 9 km/h. Determine the direction the wind appears to come from to a person standing on the ship. (3 marks)

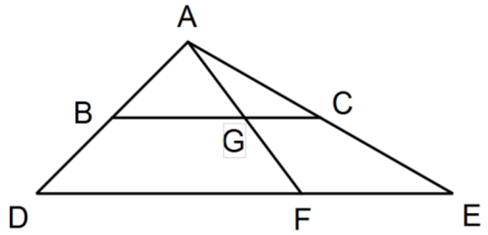
$${}_w\mathbf{v}_s = \mathbf{v}_w - \mathbf{v}_s$$



From the SW

Question 8 **(7 marks)**

In the triangle shown, F divides the side DE in the ratio $2:1$, BC is parallel to DE and G is the midpoint of AF.



Let $\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$.

(a) Show that $\overrightarrow{AG} = \frac{2}{3}\mathbf{c} + \frac{1}{3}\mathbf{b}$. (4 marks)

$$\begin{aligned} AG &= \tfrac{1}{2}AF \\ &= \tfrac{1}{2}\left(AD + DF\right) \\ &= \tfrac{1}{2}\left(AD + \tfrac{2}{3}DE\right) \\ &= \tfrac{1}{2}(2\mathbf{b} + \tfrac{2}{3}(2\mathbf{c} - 2\mathbf{b})) \\ &= \mathbf{b} + \tfrac{2}{3}\mathbf{c} - \tfrac{2}{3}\mathbf{b} \\ &= \tfrac{2}{3}\mathbf{c} + \tfrac{1}{3}\mathbf{b} \end{aligned}$$

(b) Hence, or otherwise, prove that G divides BC in the ratio $2:1$. (3 marks)

$$\begin{aligned} \overrightarrow{BG} &= \overrightarrow{BA} + \overrightarrow{AG} \\ &= -\mathbf{b} + \tfrac{2}{3}\mathbf{c} + \tfrac{1}{3}\mathbf{b} \\ &= \tfrac{2}{3}\mathbf{c} - \tfrac{2}{3}\mathbf{b} \end{aligned}$$

$$\begin{aligned} \overrightarrow{GC} &= \overrightarrow{GA} + \overrightarrow{AC} \\ &= \mathbf{c} - \left(\tfrac{2}{3}\mathbf{c} + \tfrac{1}{3}\mathbf{b}\right) \\ &= \tfrac{1}{3}\mathbf{c} - \tfrac{1}{3}\mathbf{b} \\ &= \tfrac{1}{2}\overrightarrow{BG} \end{aligned}$$

Hence G divides BC in ratio 2:1

End of questions

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